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Optimal Testing in Functional Analysis of Variance Models

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P L A N

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■ INTRODUCTION

■ *Analysis of variance* (ANOVA) - one of the most widely used tools in applied statistics. Useful for handling low dimensional data, limitations in analyzing *functional* responses.

Functional analysis of variance (FANOVA) methods provide alternatives to classical ANOVA methods while still allowing a simple interpretation.

■ General: Ramsay & Silverman (1997, 2002) and Stone *et al.* (1997).

■ Fitting and Estimation of Components: Wahba *et al.*, 1995; Stone *et al.*, 1997; Huang, 1998; Lin, 2000; Gu, 2002.

■ MODEL

■ Diffusion version of FANOVA. One observes a series of sample paths of a stochastic process driven by

$$dY_i(\mathbf{t}) = m_i(\mathbf{t})\, d\mathbf{t} + \epsilon\, dW_i(\mathbf{t}), \quad i = 1, \ldots, r; \quad \mathbf{t} \in [0,1]^d,$$

where $\epsilon > 0$ is the diffusion coefficient, r and d are finite integers, m_i are (unknown) d-dimensional response functions and W_i are independent d-dimensional standard Wiener processes.

■ Results of Brown & Low (1996): Under general conditions, the corresponding discrete model is asymptotically equivalent to the diffusion model with $\epsilon = \sigma/\sqrt{n}$.

MODEL

[Antoniadis, 1984]: Each of the r response functions in model admits the following unique decomposition

$$m_i(\mathbf{t}) = m_0 + \mu(\mathbf{t}) + a_i + \gamma_i(\mathbf{t}) \quad i = 1, \ldots, r; \quad \mathbf{t} \in [0,1]^d,$$

where m_0 is a constant function (the *grand mean*), $\mu(\mathbf{t})$ is either zero or a non-constant function of $\mathbf{t}$ (the *main effect* of $\mathbf{t}$), a_i is either zero or a non-constant function of i (the *main effect* of i) and $\gamma_i(\mathbf{t})$ is either zero or a non-zero function which cannot be decomposed as a sum of a function of i and a function of $\mathbf{t}$ (the *interaction* component).

■ IDENTIFIABILITY CONSTRAINTS

- $$\int_{[0,1]^d} \mu(\mathbf{t})\, d\mathbf{t} = 0, \quad \sum_{i=1}^{r} a_i = 0,$$

- $$\sum_{i=1}^{r} \gamma_i(\mathbf{t}) = 0, \quad \int_{[0,1]^d} \gamma_i(\mathbf{t})\, d\mathbf{t} = 0, \quad \forall\, i = 1, \dots, r; \;\; \mathbf{t} \in [0,1]^d.$$

■ Difficulties of Pointwise ANOVA

■ "Dissipation of power."

■ Fan & Lin (1998) proposed a powerful overall test for functional hypothesis testing → decomposition of the original functional data into Fourier (or wavelet) series expansions + adaptive Neyman and wavelet thresholding procedures of Fan (1996) to the resulting empirical Fourier (wavelet) coefficients.

Idea: *Sparsity* of data in non-standard (wavelet) domains.

■ Similar in Eubank (2000) and Dette & Derbort (2001).

■ Guo (2002) suggested a MLR based test for functional variance components in mixed-effects FANOVA models.

■ Optimality of Tests?

■ Not discussed in FANOVA context.

■ We derive asymptotically (as $\epsilon \to 0$ or, equivalently, as $n \to \infty$) optimal (minimax) *non-adaptive* and *adaptive* testing procedures for testing the significance of the main effect and the interactions in the FANOVA model against composite nonparametric alternatives (separated away from null in $L^2([0,1]^d)$-norm)

■ Gaussian signal + noise models: Ingster (1982, 1993), Ermakov (1990), Spokoiny (1996, 1998), Lepski & Spokoiny (1999), Ingster & Suslina (2000) and Horowitz & Spokoiny (2001)

Hypotheses to be Tested 1

Testing the significance of the main effects and the interactions

$$
\begin{aligned}
H_0 &: \mu(\mathbf{t}) \equiv 0, \qquad \mathbf{t} \in [0,1]^d, \\
H_0 &: \gamma_i(\mathbf{t}) \equiv 0, \qquad \forall\, i = 1, \ldots, r, \;\; \mathbf{t} \in [0,1]^d.
\end{aligned}
$$

Identifiability constraints $\rightarrow$

$$
Y_i^* = m_0 + a_i + \epsilon\, \xi_i, \quad i = 1, \ldots, r, \quad \sum_{i=1}^{r} a_i = 0,
$$

where $Y_i^* = \int_{[0,1]^d} dY_i(\mathbf{t})$ and ξ_i are independent $\mathcal{N}(0,1)$ random variables. This is the classical one-way fixed-effects ANOVA model.

■ Hypotheses to be Tested 2

We assume that m_i (and, hence, μ and γ_i as well) belong to a Besov ball of radius $C > 0$ on $[0,1]^d$, $B^s_{p,q}(C)$, where $s > 0$ and $1 \leq p, q \leq \infty$.

■ Interested in: **Rate** at which the distance between the null and alternative hypotheses decreases to zero, while still permitting consistent testing. Alternatives are separated away from the null by ρ in the $L^2([0,1]^d)$.

■ Alternatives are of the form

$$H_1 \quad : \quad \mu \in \mathcal{F}(\rho),$$

$$H_1 \quad : \quad \gamma_i \in \mathcal{F}(\rho), \quad \text{at least for one} \quad i = 1, \ldots, r,$$

where $\mathcal{F}(\rho) = \{f \in B^s_{p,q}(C) : ||f||_2 \geq \rho\}$.

■ Minimax Optimality 1

Consider the general model

$$dZ(\mathbf{t}) = f(\mathbf{t})\,d\mathbf{t} + \epsilon\,dW(\mathbf{t}), \quad \mathbf{t} \in [0,1]^d,$$

where W is a d-dimensional standard Wiener process.

We wish to test

$$H_0 : f \equiv 0 \quad \text{versus} \quad H_1 : f \in \mathcal{F}(\rho),$$

where $\mathcal{F}(\rho) = \{f \in B^s_{p,q}(C) : ||f||_2 \geq \rho\}$.

■ For prescribed α and β, the rate of decay to zero of the "indifference threshold" $\rho = \rho(\epsilon)$, as $\epsilon \to 0$, can be viewed as a **measure of goodness of a test. It is natural to seek the test with the optimal (fastest) rate.**

Minimax Optimality 2

[Ingster, 1993; Spokoiny, 1996; Ingster & Suslina, 2000].

Definition $\rho(\epsilon)$ is the minimax rate of testing if $\rho(\epsilon) \to 0$ as $\epsilon \to 0$ and the following two conditions hold

(i) for any $\rho'(\epsilon)$ satisfying $\rho'(\epsilon)/\rho(\epsilon) = o_\epsilon(1)$, one has

$$\inf_{\phi_\epsilon} \left[\alpha(\phi_\epsilon) + \beta(\phi_\epsilon, \rho'(\epsilon))\right] = 1 - o_\epsilon(1),$$

where $o_\epsilon(1) \to 0$ as $\epsilon \to 0$.

(ii) for any $\alpha > 0$ and $\beta > 0$ there exists a constant $c > 0$ and a test ϕ_ϵ^* such that

$$\alpha(\phi_\epsilon^*) \leq \alpha + o_\epsilon(1), \quad \beta(\phi_\epsilon^*, c\rho(\epsilon)) \leq \beta + o_\epsilon(1).$$

ϕ_ϵ^* is called an asymptotically optimal (minimax) test.

■ Minimax Optimality 3

■ Ingster (1993) and Lepski & Spokoiny (1999) showed that for $sp > d$ the asymptotically optimal (minimax) rate is

$$\rho(\epsilon) = \epsilon^{4s''/(4s''+d)},$$

where $s'' = \min(s, s - \frac{d}{2p} + \frac{d}{4})$.

■ The proposed asymptotically optimal (minimax) tests were consistent but *non-adaptive* [involve the smoothness parameters s and p of the corresponding Besov ball].

■ Spokoiny (1996) and Horowitz & Spokoiny (2001): Problem of *adaptive* minimax testing where s and p are unknown. **No adaptive test can achieve the exact optimal rate uniformly over all s and p** (in some given range).

■ Minimax Optimality 4

■ Price for Adaptivity: If one allows increase of $\rho(\epsilon)$ by an additional log-log factor $t_\epsilon = (\ln\ln \epsilon^{-2})^{1/4}$, i.e, considers $\rho(\epsilon t_\epsilon)$ instead of $\rho(\epsilon)$, then [Horowitz & Spokoiny (2001)] the optimal rate of adaptive testing is

$$\rho(\epsilon t_\epsilon) = (\epsilon t_\epsilon)^{4s''/(4s''+d)},$$

■ The "price" factor t_ϵ is unavoidable and cannot be reduced.

Wavelet Bases

We assume $d = 1$ and work with periodic o.n. wavelet bases in $L^2([0,1])$ generated by shifts of a compactly supported scaling function ϕ, i.e.

$$\phi^{\mathrm{p}}(t) = \sum_{\ell\in\mathbb{Z}} \phi(t-\ell),\ \psi^{\mathrm{p}}_{jk}(t) = \sum_{\ell\in\mathbb{Z}} \psi_{jk}(t-\ell),\ j \geq 0, k = 0, \ldots, 2^j - 1$$

where

$\psi_{jk}(t) = 2^{j/2}\psi(2^j t - k)$. $\{\phi^{\mathrm{p}};\ \psi^{\mathrm{p}}_{jk},\ j \geq 0,\ k = 0, 1, \ldots, 2^j - 1\}$ generates an o.n. basis in $L^2([0,1])$.

If the MRA is of regularity $r > 0$, the corresponding wavelet basis is **unconditional** for Besov spaces $B^s_{p,q}([0,1])$ for $0 < s < r$, $1 \leq p, q \leq \infty$. Such bases characterize Besov balls in terms of wavelet coefficients.

Testing in FANOVA 1

Averaging over r paths + identifiability conditions:

$$\begin{aligned} d\overline{Y}(t) &= (m_0 + \mu(t))\,dt + \epsilon\,d\overline{W}(t),\ \ t \in [0,1] \\ d(Y_i - \overline{Y})(t) &= (a_i + \gamma_i(t))\,dt + \epsilon\,d(W_i - \overline{W})(t),\ \ i = 1,\dots,r. \end{aligned}$$

$\{W_i - \overline{W};\ i = 1,\dots,r\}$ are Wiener processes with the same covariance kernel $C(s,t) = \frac{r-1}{r}\min(s,t)$ [but no independent].

$$dZ(t) = f(t)\,dt + \eta\,dW(t),\ \ t \in [0,1],$$

- $Z(t) = \overline{Y}(t)$, $f(t) = m_0 + \mu(t)$, $\eta = \epsilon/\sqrt{r}$
- $Z(t) = (Y_i - \overline{Y})(t)$, $f(t) = a_i + \gamma_i(t)$, $\eta = \epsilon\sqrt{(r-1)/r}$

■ Testing in FANOVA 2

To apply Spokoiny (1996) results, assume that $B^s_{p,q}(C)$ satisfies $1 \le p, q \le \infty$, $sp > 1$ and $s - \frac{1}{2p} + \frac{1}{4} > 0$. [Donoho *et al.*, 1995; Donoho & Johnstone, 1998)].

$$H_0 : f \equiv \text{constant } \left(= \int_0^1 f(t)dt \right)$$

versus

$$H_1 : \left(f - \int_0^1 f(t)dt \right) \in \mathcal{F}(\rho),$$

where $\mathcal{F}(\rho) = \{f \in B^s_{p,q}(C) : ||f||_2 \ge \rho\}$, $1 \le p, q \le \infty$, $sp > 1$ and $s - \frac{1}{2p} + \frac{1}{4} > 0$.

■ Testing in FANOVA 3

■ Choose a wavelet ψ of regularity $r > s$. One has

$$Y_{jk} = \theta_{jk} + \eta\, \xi_{jk}, \quad j \geq -1; \quad k = 0, 1, \ldots, 2^j - 1,$$

where $Y_{jk} = \int_0^1 \psi_{jk}(t) dZ(t)$, $\theta_{jk} = \int_0^1 \psi_{jk}(t) f(t) dt$ and ξ_{jk} are independent $\mathcal{N}(0, 1)$ random variables.

■ Testing

$$H_0 : \; f \equiv \text{constant}$$

is equivalent to testing

$$H_0 : \; \theta_{jk} = 0 \quad \forall\, j \geq 0; \quad k = 0, 1, \ldots, 2^j - 1.$$

■ NON-ADAPTIVE TEST 1

RESULT Let the MRA be of regularity $r > s$, and let the parameters s, p, q and the radius C of the Besov ball $B^s_{p,q}(C)$ be **known,** where $1 \le p, q \le \infty$, $sp > 1$, $s - \frac{1}{2p} + \frac{1}{4} > 0$ and $C > 0$. Then, for a fixed significance level $\alpha \in (0,1)$, the test ϕ^*, for testing

$$H_0 : f \equiv \text{constant} \quad \text{vs} \quad H_1 : \left(f - \int_0^1 f(t)dt \right) \in \mathcal{F}(\rho),$$

where $\mathcal{F}(\rho) = \{f \in B^s_{p,q}(C) : ||f||_2 \ge \rho\}$, is α-level asymptotically optimal (minimax) test, as $\eta \to 0$. That is, for any $\beta \in (0,1)$, it attains the optimal rate of testing

$$\rho(\eta) = \eta^{4s''/(4s''+1)},$$

where $s'' = \min\{s, s - \frac{1}{2p} + \frac{1}{4}\}$.

■ NON-ADAPTIVE TEST 2

- ϕ^* is based on the sum of squares of the thresholded empirical wavelet coefficients Y_{jk} with properly chosen level-dependent thresholds. The null hypothesis is rejected when this sum of squares exceeds some critical value.
- j_η the largest integer: $j_\eta \leq \log_2 \eta^{-2}$.
- $j(s)$ resolution level given by

$$j(s) = \frac{2}{4s'' + 1} \log_2 \left(C\eta^{-2} \right).$$

- Levels split as:

$$\mathcal{J}_- = \{0, \ldots, j(s) - 1\}, \quad \mathcal{J}_+ = \{j(s), \ldots, j_\eta - 1\}.$$

NON-ADAPTIVE TEST 3

- For each $j \in \mathcal{J}_-$, define

$$S_j = \sum_{k=0}^{2^j-1} (Y_{jk}^2 - \eta^2)$$

- For each $j \in \mathcal{J}_+$ and for given threshold $\lambda > 0$, define

$$S_j(\lambda) = \sum_{k=0}^{2^j-1} [(Y_{jk}^2 \mathbf{1}(|Y_{jk}| > \eta\lambda) - \eta^2 b(\lambda)],$$

where $b(\lambda) = \mathbb{E}\left[\xi^2 \mathbf{1}(|\xi| > \lambda)\right]$ and $\xi \sim \mathcal{N}(0, 1)$.

■ NONADAPTIVE TEST 4

■ Define

$$T(j(s)) = \sum_{j=0}^{j(s)-1} S_j,$$

and

$$Q(j(s)) = \sum_{j=j(s)}^{j_\eta - 1} S_j(\lambda_j),$$

where $\lambda_j = 4\sqrt{(j - j(s) + 8)\ln 2}$.

■ Under H_0,

$v_0^2(j(s)) = 2\eta^4 2^{j(s)}$ and $w_0^2(j(s)) = \eta^4 \sum_{j=j(s)}^{j_\eta - 1} 2^j d(\lambda_j)$, are the variances of $T(j(s))$ and $Q(j(s))$, respectively, where $d(\lambda_j) = \mathbb{E}\left[\xi^4 \mathbf{1}(|\xi| > \lambda_j)\right]$.

■ NON-ADAPTIVE TEST 5

Comment: In MATLAB simulations we replaced the expression from Fan (1996):

$$\begin{aligned} \mathbb{E}(\xi^{2k}\mathbf{1}(|\xi| > \lambda_j)) &= \sqrt{2/\pi}\lambda_j^{2k-1}2^{-8(j-j(s)+8)} \\ &+ O\left(\lambda_j^{2k-3}2^{-8(j-j(s)+8)}\right), \quad k = 1, 2, \ldots . \end{aligned}$$

by

$$d(\lambda_j) = 3 - \sqrt{2/\pi}\Lambda_j^5/5 + \Lambda_j^7/(7\sqrt{2\pi}) + o(\Lambda_j^8),$$

where $\Lambda_j = \min(\lambda_j, 1/\lambda_j)$. Similar approximation can be derived for $b(\lambda_j) = \mathbb{E}(\xi^2\mathbf{1}(|\xi| > \lambda_j)$.

■ NON-ADAPTIVE TEST 6

■ Finally, for a given significance level $\alpha \in (0,1)$, let ϕ^* be the test defined by

$$\phi^* = \begin{cases} \mathbf{1}\left\{T(j(s)) > v_0(j(s))z_{1-\alpha}\right\}, & \text{if } p \geq 2 \\ \mathbf{1}\left\{T(j(s)) + Q(j(s)) > \sqrt{v_0^2(j(s)) + w_0^2(j(s))}z_{1-\alpha}\right\}, & \\ \qquad \text{if } 1 \leq p < 2, & \end{cases}$$

■ ADAPTIVE TEST 1

■ The parameters s, p, q and the radius C of the corresponding Besov ball $B^s_{p,q}(C)$ are unknown. Assume that $0 < s \leq s_{\max}$, $1 \leq p, q \leq \infty$, $sp > 1$, $s - \frac{1}{2p} + \frac{1}{4} > 0$ and $0 < C \leq C_{max}$.

■ Let $t_\eta = (\ln \ln \eta^{-2})^{1/4}$ and $j_{\min} = \frac{2}{4s_{\max}+1} \log_2 \eta^{-2}$.

■ Regularity of MRA: $r > s_{\max}$.

■ The idea: Consider the range of $j(s) = j_{\min}, \ldots, j_\eta - 1$ and reject H_0 if it is rejected at least for one selected level $j(s)$.

■ ADAPTIVE TEST 2

Since $\mathrm{card}(\{j_{\min}, \ldots, j_\eta - 1\}) = O(\ln \eta^{-2})$, Bonferroni type testing leads to the asymptotically *adaptive* test

$$\phi_\eta^* = \mathbf{1}\left[\max_{j_{\min} \le j(s) \le j_\eta - 1} \left\{ \frac{T(j(s)) + Q(j(s))}{\sqrt{v_0^2(j(s)) + w_0^2(j(s))}} \right\} > \sqrt{2 \ln \ln \eta^{-2}} \right].$$

■ ADAPTIVE TEST 3

Spokoiny (1996) showed that the test ϕ^*_η is an adaptive optimal test, i.e.

$$\alpha(\phi^*_\eta) = o_\eta(1)$$

and

$$\sup_{\mathcal{T}} \beta(\phi^*_\eta, c\rho(\eta t_\eta)) = o_\eta(1),$$

where $\rho(\eta t_\eta) = (\eta t_\eta)^{4s''/(4s''+1)}$, $o_\eta(1) \to 0$ as $\eta \to 0$, and c is a constant.

If it is known that $p \geq 2$ then the adaptive test can be simplified to

$$\phi^*_\eta = \mathbf{1}\left[\max_{j_{\min} \leq j(s) \leq j_\eta - 1} \left\{ \frac{T(j(s))}{\sqrt{v_0^2(j(s))}} \right\} > \sqrt{2 \ln \ln \eta^{-2}}\right].$$

■ A COMMENT

The test ϕ^*_η is similar in spirit to that in Fan (1996) and Fan & Lin (2000), though they apply a *global* threshold.

■ APPLICATIONS

■ SIMULATION STUDY 1

■ Synthetic data from the battery of standard test functions of Donoho & Johnstone (1995): BLOCKS, BUMPS, DOPPLER and HEAVISINE. Additional test function MISHMASH, defined as

$$\text{MISHMASH} = -(\text{ BLOCKS} + \text{ BUMPS} + \text{DOPPLER} + \text{HEAVISINE}),$$

added because of the identifiability constraints.

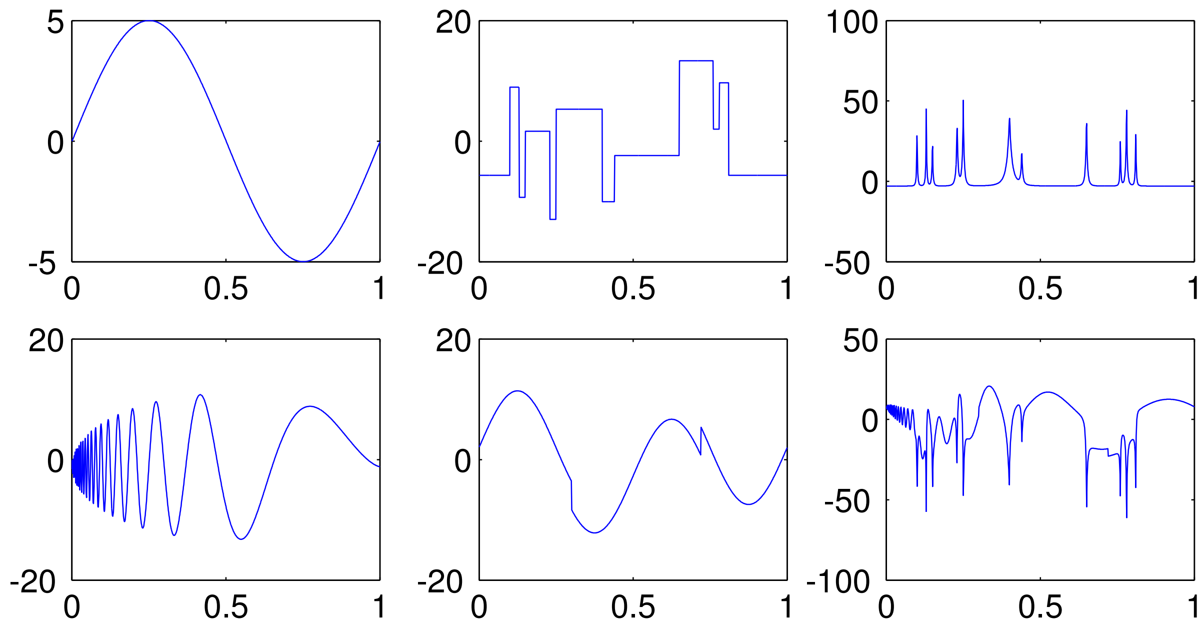

Figure 1: The mean function $\mu(t) = 5\sin(2\pi t)$ and the centered treatment effect functions $\gamma_i(t)$, $i = 1, \ldots, 5$ (i.e., centered BLOCKS, BUMPS, DOPPLER, HEAVISINE, and MISHMASH), sampled at $n = 1024$ data points.

■ SIMULATION STUDY 3

■ $m_0 = 1,\ \mu(t) = 5\sin(2\pi t)$

■ Five simulated observations (one for each test function shown; length ($n = 1024$), two SNRs (SNR = 3 and 7).

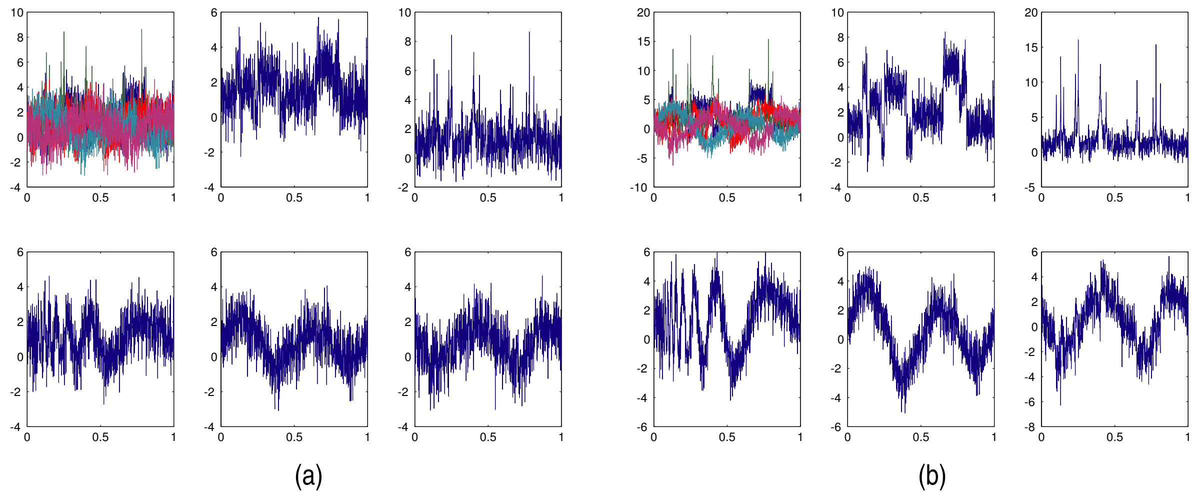

Figure 2: Five simulated observations (one for each test function shown in Figure 1) sampled at $n = 1024$ data points are shown superimposed (first plot) and separately (remaining five plots) for (a) SNR = 3 and (b) SNR = 7.

■ SIMULATION STUDY 4

■ To test the hypothesis $H_0 : \mu(t) = 0$, nonadaptive test, $p \geq 2$. ■ Symmlet 8-tap

■ $j(s) = 3$

■ SNR=3: $T(3) = 15.28$ critical value 1.5949

■ SNR=7: $T(3) = 97.52$ critical value 1.6316.

■ $H_0 : \gamma_i(t) = 0$ $(i = 1, \ldots, 5)$, non-adaptive test, $1 \leq p < 2$. ■ Daubechies 6-tap

■ $j(s) = 3$ ■ $j_\eta = 7$.

■ SNR=3, $T(3) + Q(3) = 275.3326$ critical value 154.6294

■ SNR=7, $T(3) + Q(3) = 5941.099$ critical value 156.4943

■ SIMULATION STUDY 5

■ Extensive power analysis for the above tests against the composite alternatives

$$H_1 : \mu \in \mathcal{F}(\rho) \quad \text{and} \quad H_1 : \frac{1}{5}\sum_{i=1}^{5} \gamma_i \in \mathcal{F}(\rho). \tag{1}$$

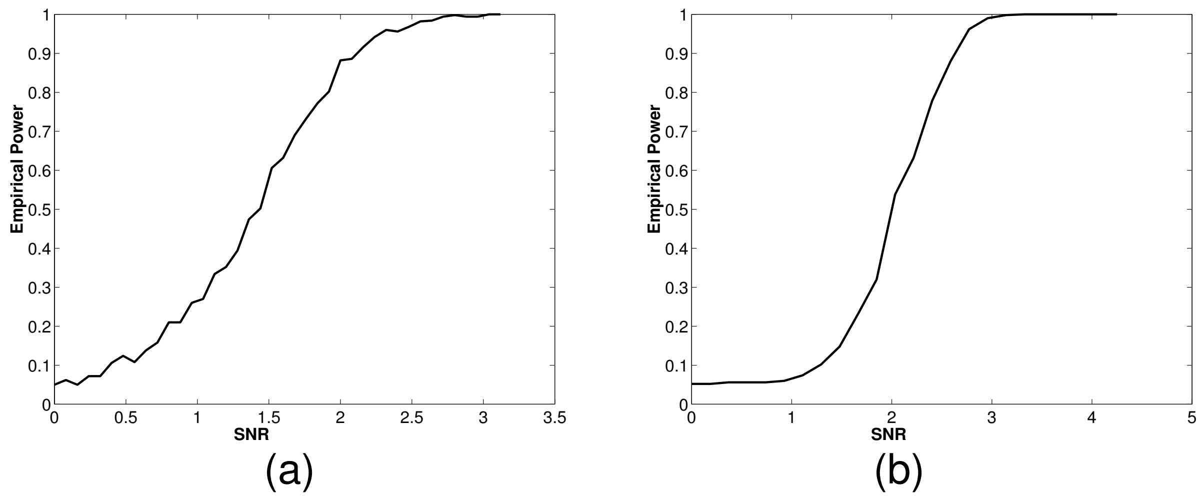

Figure 3: Empirical power functions for testing (a) $H_0 : \mu(t) = 0$ versus $H_1 : ||\mu||_2 = \rho$ and (b) $H_0 : \gamma_i(t) = 0$ $(i = 1, \ldots, 5)$ versus $H_1 : ||\sum_i \gamma_i/5||_2 = \rho$. In both panels, the sample size was $n = 512$ and the number of trials at a fixed discretized SNR was 500.

■ ORTHOSIS DATA ANALYSIS 1

- Interesting data on human movement.
- Data: Amarantini David and Martin Luc, Laboratoire Sport et Performance Motrice, Grenoble University
- Underlying movement under various levels of an externally applied force to the knee.
- Seven young male volunteers wore a spring-loaded orthosis of adjustable stiffness under 4 experimental conditions:
  - Control condition (without orthosis),
  - Orthosis condition,
  - Two conditions (Spring1, Spring2) stepping in place was perturbed by fitting a spring-loaded orthosis onto the right knee.

■ ORTHOSIS DATA ANALYSIS 2

■ The data set consists in 280 separate runs and involves the seven subjects over four described experimental conditions, replicated ten times for each subject.

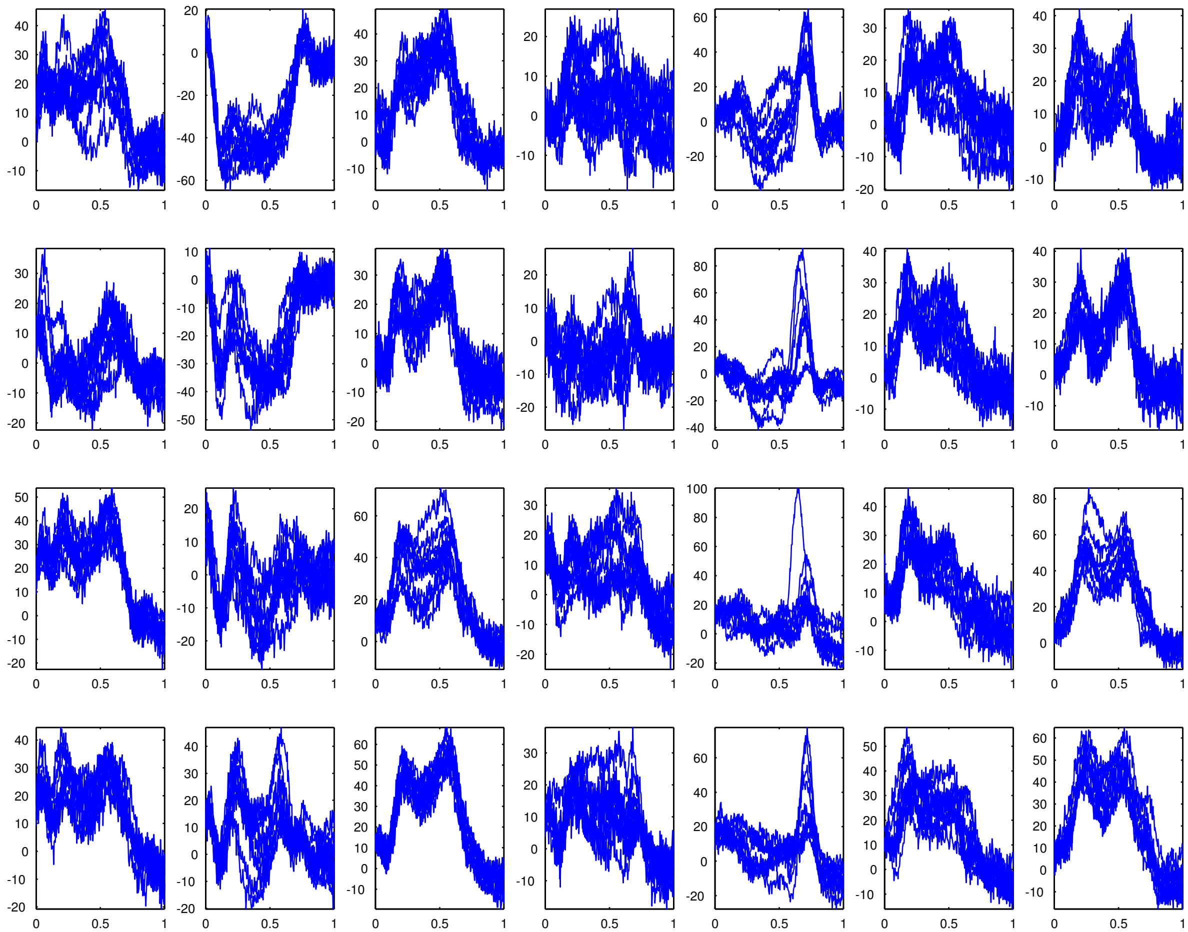

Figure 4: Orthosis data set: panels in rows correspond to *Treatments* while the panels in columns correspond to *Subjects*.

■ ORTHOSIS DATA ANALYSIS 3: MODEL

■ Model

$$dY_{ijk}(t) = m_{ij}(t)\,dt + \epsilon\,dW_{ijk}(t),$$
$$i = 1,\dots,I;\ j = 1,\dots,J;\ k = 1,\dots,K;\ t \in [0,1],$$

with

$$m_{ij}(t) = m_0 + \mu(t) + \alpha_i + \gamma_i(t) + \beta_j + \delta_j(t),$$
$$i = 1,\dots,I;\ j = 1,\dots,J;\ t \in [0,1],$$

where i is the condition index, j is the subject index, k is the replication index, and t is the time.

■ Subjects in the above model are naturally considered as **block effects**; subjects obviously differ but the researchers are not interested in their differences.

■ ORTHOSIS DATA ANALYSIS 4: MODEL

$$d\bar{Y}_{i\cdot\cdot}(t) = m_i(t)\,dt + \eta\,dW_{i\cdot\cdot}(t),\ i = 1,\ldots,I;\ t \in [0,1],$$

with

$$m_i(t) = m_0 + \mu(t) + \alpha_i + \gamma_i(t),\ i = 1,\ldots,I;\ t \in [0,1],$$

where $\eta = \epsilon/\sqrt{JK}$.

- $j(s) = 4$ and $j_\eta = 6$.
- *Coiflet 18-tap filter*

ORTHOSIS DATA ANALYSIS 5

- Tests $H_0 : \mu(t) = 0$ and $H_0 : \gamma_i = 0$ were both significant.
- The researchers interested contrasts:
- Control and Orthosis functional treatment effects are equal ($H_0 : \gamma_1(t) = \gamma_2(t)$). Not significant, p-value 0.157
- Spring 1 and Spring 2 functional treatment effects are equal ($H_0 : \gamma_3(t) = \gamma_4(t)$). Not significant, p-value 0.198.
- Contrast $(\gamma_1(t) + \gamma_2(t)) - (\gamma_3(t) + \gamma_4(t))$. Significant, p-value is 0.0103.

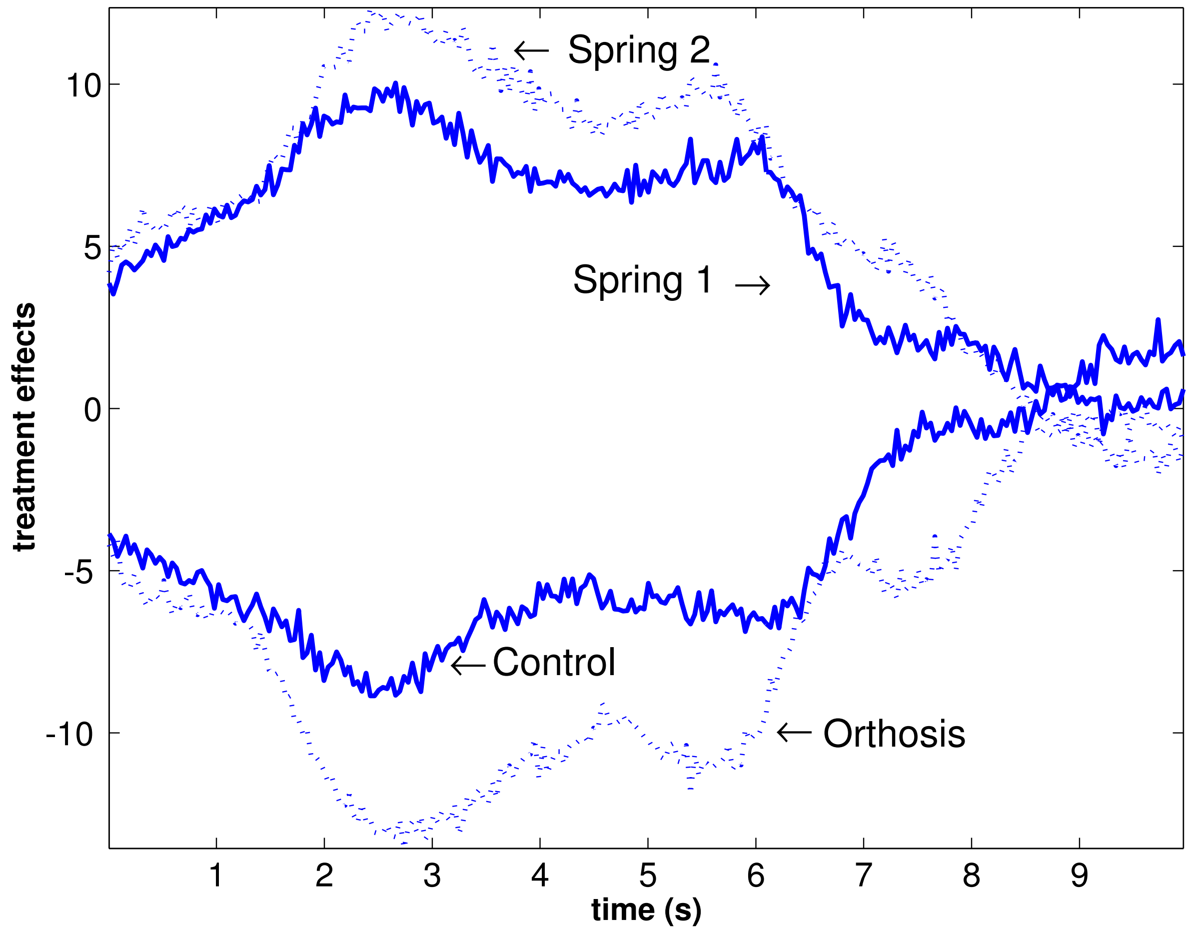

Figure 5: Empirical estimators of the treatment effects of interest. Constant and functional components α_i and $\gamma_i(t)$ $(i = 1, \ldots, 4)$ are not separated.

■ CONCLUSIONS

■ $dY(s,t) =$
$(m_0 + a(s) + \mu(t) + \gamma(s,t))\, dt\, ds + \epsilon\, dW(s,t), \quad (s,t) \in [0,1]^2$

■ $d \geq 2$, [Thresholding? Block Thresholding, FDR?]

■ Black Box Procedure: Variances of T, S by bootstrap [wavestrap, Percival et al. 1999].

■ Data, Matlab Files: `brani@isye.gatech.edu`.