# Historical Highlights
# in the Development of Categorical Data Analysis


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# Karl Pearson (1857-1936)

## Karl Pearson (1900) *Philos. Mag.*

Introduces chi-squared statistic

$$X^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

$$df = \text{no. categories} - 1$$

- testing values for multinomial probabilities

  (Monte Carlo roulette runs)

- testing fit of Pearson curves
- testing statistical independence in $r \times c$ contingency table $(df = rc - 1)$

# Karl Pearson (1904)

Advocates measuring association in contingency tables by approximating the correlation for an assumed underlying continuous distribution

- tetrachoric correlation ($2 \times 2$, assuming bivariate normality)
- contingency coefficient $\sqrt{\frac{X^2}{X^2+n}}$ based on $X^2$ for testing independence in $r \times c$ contingency table
- introduces term “contingency” as a “measure of the total deviation of the classification from independent probability.”

# George Udny Yule (1871-1951)

(1900) *Philos. Trans. Royal Soc. London*

(1912) *JRSS*

Advocates measuring association using odds ratio

| | |
|---|---|
| $n_{11}$ | $n_{12}$ |
| $n_{21}$ | $n_{22}$ |

$$\hat{\theta} = \frac{n_{11}n_{22}}{n_{12}n_{21}} \qquad Q = \frac{n_{11}n_{22} - n_{12}n_{21}}{n_{11}n_{22} + n_{12}n_{21}} = (\hat{\theta} - 1)/(\hat{\theta} + 1)$$

“At best the normal coefficient can only be said to give us. . . a hypothetical correlation between supposititious variables. The introduction of needless and unverifiable hypotheses does not appear to me a desirable proceeding in scientific work.”

(1911) *An Introduction to the Theory of Statistics* (14 editions)

# K. Pearson, with D. Heron (1913) *Biometrika*

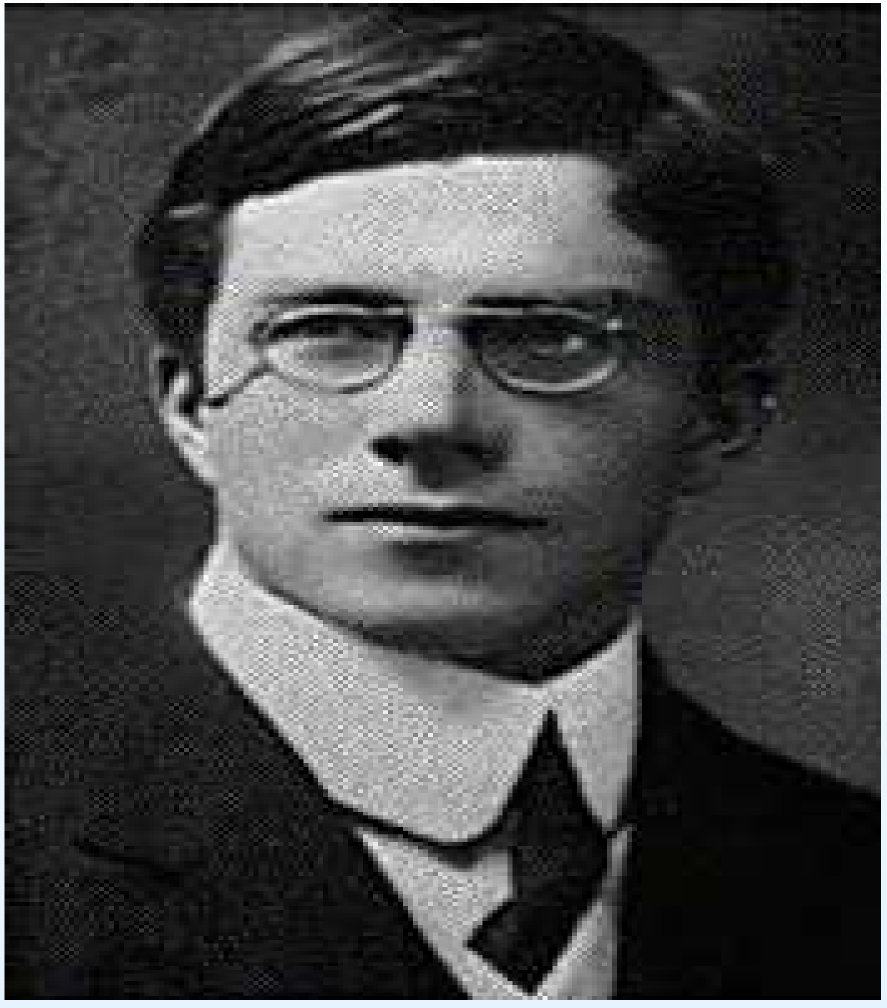
"Unthinking praise has been bestowed on a textbook which can only lead statistical students hopelessly astray."

⋮

"If Mr. Yule's views are accepted, irreparable damage will be done to the growth of modern statistical theory . . . Yule's $Q$ has never been and never will be used in any works done under my supervision. . . . Yule must withdraw his ideas if he wishes to maintain any reputation as a statistician."

and so on, for 150 pages

# Ronald A. Fisher (1890-1962)



## R. A. Fisher (1922)

- Introduces concept of degrees of freedom with geometrical argument.
- Shows that when marginal proportions in $r \times c$ table are estimated, the additional $(r-1)+(c-1)$ constraints imply

$$df = (rc-1) - [(r-1)+(c-1)] = (r-1)(c-1)$$

K. Pearson (1922)

“Such a view is entirely erroneous. The writer has done no service to the science of statistics by giving it broad-cast circulation in the pages of JRSS. I trust my critic will pardon me for comparing him with Don Quixote tilting at the windmill; he must either destroy himself, or the whole theory of probable errors, for they are invariably based on using sample values for those of the sampled population unknown to us.”

# Fisher uses data from Pearson's son Egon

E. S. Pearson (1925) *Biometrika* generated $>$ 12,000 "random" $2 \times 2$ tables, for paper about Bayes Theorem

$df$ = 3 or $df$ = 1?

Fisher (1926) *Eugenics Rev.*

$$\frac{\sum_{i=1}^{12,000} X_i^2}{12,000} = 1.00001$$

In a later volume of his collected works (1950), Fisher wrote of Pearson, "If peevish intolerance of free opinion in others is a sign of senility, it is one which he had developed at an early age."

# Fisher's exact test

2nd ed. *Statistical Methods for Research Workers* (1934),
*The Design of Experiments* (1935)

Tea-tasting lady: Dr. Muriel Bristol, Rothamsted

| | | GUESS | | |
|---|---|---|---|---|
| | | Milk | Tea | |
| ACTUAL | Milk | 3 | 1 | 4 |
| Poured first | Tea | 1 | 3 | 4 |
| | | 4 | 4 | |

$$P\text{-value} = \frac{\binom{4}{3}\binom{4}{1} + \binom{4}{4}\binom{4}{0}}{\binom{8}{4}} = 0.243$$

## Fisher (1936) *Annals of Science*

Analyzes data from Mendel (1865)
(experiments testing theories of natural inheritance)
For 84 separate $2 \times 2$ tables, $\sum X^2 = 42$ ($df$ = 84)

$$P_{H_0}(\chi^2_{84} \leq 42) = 0.00004$$

"When data have been faked, ... people underestimate the frequency of wide chance deviations; the tendency is always to make them agree too well with expectations. The data of most, if not all, of the experiments have been falsified so as to agree closely with Mendel's expectations."

Fisher also proposed *partitioning chi-squared* (SMRW), and used canonical correlation to assign scores to rows and columns of contingency table to maximize correlation (1940), which relates to later *correspondence analysis* methods.

## Maurice Bartlett (1935 *JRSS*)

For probabilities in a $2 \times 2 \times 2$ cross-classification of $(X, Y, Z)$, “no interaction” defined as identical $XY$ odds ratio at each level of $Z$

Shows maximum likelihood (ML) approach to estimating cell probabilities satisfying this condition

(attributes idea to R.A. Fisher)

End of story? Lancaster (1951 *JRSS-B*) “Doubtless little use will ever be made of more than a three-dimensional classification.”

# The *probit* model for binary data

Chester Bliss: (1934) *Science*

(1935) *Ann. Appl. Biol.*

Popularizes probit model for applications in toxicology

Binary $y$ with $y = 1$ for death, $x$ = dosage or log dosage.
Underlying latent variable model for tolerance implies

Model: $P(y = 1) = \Phi(\alpha + \beta x)$

for *cdf* $\Phi$ of $N(0, 1)$ r.v.

R. A. Fisher (appendix to Bliss 1935) provides “Fisher scoring” algorithm for ML fitting of probit model

## The *logit*

- Maurice Bartlett (1937 *JRSS*) uses $\log[y/(1-y)]$ to transform continuous proportions for use in regression and ANOVA

- R. A. Fisher and Frank Yates (1938) *Statistical Tables* suggest transformation $\log\left[\frac{P(y=1)}{P(y=0)}\right]$ of binomial parameter

- Joseph Berkson (1944 *JASA*) of Mayo Clinic introduces term *logit* for $\log\left[\frac{P(y=1)}{P(y=0)}\right]$, shows similarity in shape of probit model and logistic regression model
  $P(y=1) = F(\alpha + \beta x)$ for logistic *cdf* $F(z) = \frac{e^z}{1+e^z}$

- D. R. Cox - Influential paper (1958 *JRSS-B*) and book (*Analysis of Binary Data*, 1970) on logistic regression

# Later advances using logistic regression

- Case-control (Cornfield 1951, Mantel 1973, Prentice 1976)
- *Ordinal* data: McKelvey and Zavoina (1975) probit for cumulative probabilities, P. McCullagh (1980) arbitrary link

  *Nominal* data: Substantial econometric literature on baseline-category logit models and related discrete choice models (Theil 1970, McFadden 1974 – Nobel prize in 2000)
- Conditional logistic regression to eliminate nuisance parameters (Breslow, Prentice and others in late 1970s)
- Cyrus Mehta and Nitin Patel (1983) Develop network algorithm for exact conditional logistic regression
- Marginal models for clustered data (GEE approach: Kung-Yee Liang and Scott Zeger 1986)
- Random effects models: Don Pierce and B. R. Sands (1975), Norman Breslow and David Clayton (1993)

# Jerzy Neyman (1949) *Berkeley symposium*

Cell probabilities $\{p_i\}$
Sample proportions $\{\hat{p}_i\}$
Model: $p_i = p_i(\theta)$

Develops BAN theory for estimators such as

- minimum chi-squared

$$\tilde{\theta} \text{ that minimizes } \sum_i \frac{(\hat{p}_i - p_i(\theta))^2}{p_i(\theta)}$$

- minimum modified chi-squared

$$\tilde{\theta} \text{ that minimizes } \sum_i \frac{(\hat{p}_i - p_i(\theta))^2}{\hat{p}_i}$$

Only mention of Fisher is disparaging comment that Fisher had claimed (not very clearly) that only ML estimators could be asymptotically efficient.

## William Cochran (1909-1980)

(1940) *Annals*: ANOVA for Poisson and binomial responses

(1943) *JASA*: Dealing with overdispersion

(1950) *Biometrika*: Cochran's $Q$ for comparing proportions in several matched samples, generalizes McNemar (1947) test

(1954) *Biometrics*: Methods for strengthening $\chi^2$

- Guidelines on using $X^2$ for small $n$
(don't need all expected frequencies $\geq$ 5)
- Partitioning $X^2$, such as a $df = 1$ test for a linear trend in proportions in a $r \times 2$ table with ordered rows
(Cochran - Armitage test)
- Test of $XY$ conditional independence in $2 \times 2 \times K$ tables
Compare $\sum_k n_{11k}$ to $E_{H_0}(\sum_k n_{11k})$, $df$ = 1
(similar to Mantel - Haenszel (1959) test)

# “Simpson’s paradox”

Edward H. Simpson, student of Bartlett (1951 *JRSS-B*)

ex Admissions to graduate school (Berkeley)

| DEPARTMENT | MALES | FEMALES |
|---|---|---|
| A | $\frac{512}{825}$ (62%) | $\frac{89}{108}$ (82%) |
| B | $\frac{22}{373}$ (6%) | $\frac{24}{341}$ (7%) |
| Total | $\frac{534}{1198}$ (45%) | $\frac{113}{449}$ (25%) |

i.e.,can be misleading to collapse contingency tables (as recommended, e.g., by Snedecor when no 3-factor interaction). Simpson proved sufficient conditions for collapsibility

Yule (1903 *Biometrika*) had noted the paradox

# Goodman and Kruskal measures of association

Leo Goodman and William Kruskal
(1954, 1959, 1963, 1972 *JASA*)

> Introduce measures of association for contingency tables, emphasize interpretability of proportional reduction in error (PRE) measures

e.g. for ordinal classifications, discrete version of Kendall's tau for concordant (C) and discordant pairs (D)

$$\hat{\gamma} = \frac{C - D}{C + D}$$

Later extensions by social scientists to ordinal models predicting sign($y_i - y_j$) using sign($x_i - x_j$) for pairs $(i, j)$ of observations

# ML estimation in contingency tables

M. W. Birch (1963) *JRSS-B*, (1964) *Annals*

- ML estimation of cell probabilities in three-way tables, under various conditions
- For general model $p_i = p_i(\theta)$, derived asymptotic dist. of $\hat{\theta}$, extending C.R. Rao (1957, 1958), H. Cramér (1946)
- Equivalence of ML estimation for Poisson and multinomial sampling
- Related work on estimation under various interaction structures by S. Roy and students (1956), J. Darroch (1962), I.J. Good (1963), and N. Mantel (1966).
- Stimulus for research the next ten years on loglinear models.

## Loglinear models

Birch, Leo Goodman, and others make explicit loglinear model formulation of multiplicative categorical data relationships

(multiplicative relationships, so log transform yields linearity, ANOVA-like models)

ex. Statistical independence of $X$ and $Y$

$$\begin{aligned} P(X=i,\ Y=j) &= P(X=i)P(Y=j) \\ \log P(X=i,\ Y=j) &= \log P(X=i) + \log P(Y=j) \\ &= \alpha_i + \beta_j \end{aligned}$$

ex. Conditional independence models in multiway tables

Darroch, Lauritzen, and Speed (1980 *Annals Statist.*) later showed graphical modeling connections

# Loglinear models (more)

Rapid advances in loglinear methodology during late 60's and early 70's at

| **Chicago** | **Harvard** | **N. Carolina** |
| --- | --- | --- |
| Leo Goodman | students of | Gary Koch |
| Shelby Haberman | F. Mosteller and | and colleagues |
| | W. Cochran | |

**Chicago**

Haberman Ph.D. thesis (1970) - outstanding theoretical development of loglinear models

## Chicago: Leo Goodman

Tremendous contributions to loglinear methodology (and related logit models for contingency tables) starting in 1964

| | |
|---|---|
| (1968, 1970) *JASA*: | Good surveys |
| | Fisher memorial lecture |
| | “quasi independence” |
| (1971) *Technometrics*: | Model-building, stepwise procedures |
| (1974) *Biometrika*: | Latent class model |
| | EM fitting, extends Lazarsfeld |
| (1979) *JASA*: | Association models for ordinal variables |
| (1986) *Int. Statist. Rev.*: | Inference for correspondence analysis |

Simultaneously, applications articles in social science journals

# Harvard: Fred Mosteller

ASA presidential address (1968) *JASA*

"I fear that the first act of most social scientists upon seeing a contingency table is to compute chi-square for it."

Paper describes influential work at this time by students of Mosteller (e.g., Bishop, Fienberg) and Cochran at Harvard

National Halothane Study

> (Is halothane more likely than other anesthetics to cause death due to liver damage?)

impetus for Bishop, Fienberg, and Holland (1975)
*Discrete Multivariate Analysis*

Several articles on loglinear models by these authors in early 1970s

# U. North Carolina: The “GSK method”

Grizzle, Starmer and Koch (1969) *Biometrics*

Apply weighted least squares methods to logit, loglinear, and other regression-type models for categorical data

Later papers by Gary Koch and students applying WLS to variety of problems, such as models for repeated categorical measurement (*Biometrics* 1977)

Vasant Bhapkar (1966) *JASA*

When model can be specified by constraint equations that are linear in $\{p_i\}$, WLS estimator = Neyman’s minimum modified chi-squared estimator

## Generalized linear models

John Nelder and Robert Wedderburn (1972) *JRSS-A*

Logistic, loglinear, probit models are special cases of generalized linear models for exponential family response distributions using various “link functions.”

- Unites categorical methods with ANOVA and regression methods for normally distributed responses
- ML fitting of all models achieved by same Fisher scoring algorithm, which is iterative WLS (GLIM)
- Wedderburn (1974) generalized to *quasi likelihood*

# Bayesian approaches for categorical data analysis

- Using beta distribution (especially, uniform) as prior for binomial goes back to Bayes (1763), Laplace (1774)
- I. J. Good (1956, 1965, et al.) smooths proportions in sparse contingency tables using Dirichlet prior (outgrowth of intelligence work during WWII at Bletchley Park with Turing), also uses empirical Bayes and hierarchical approaches
- Pat Altham (1969) considers Bayesian analyses for $2 \times 2$ tables and shows connections with frequentist results
- Steve Fienberg and Paul Holland (1970, 1973) use empirical Bayes to smooth tables
- Tom Leonard (1970s) et al. generalize Dennis Lindley's work using normal prior dist's for logit, loglinear parameters
- Arnold Zellner and Peter Rossi (1984) and later papers use simulation methods to fit binary regression models

## Some contributions by UF-WW participants: Jim Albert

- Going beyond conjugate Dirichlet priors to describe prior beliefs about association in a two-way contingency table in a hierarchical manner (1982, 1983 *Ann. Statist.*, *JRSS-B*)
- Following I.J. Good, various Bayesian ways of smoothing tables (e.g., 1987 *J. Statist. Comput. Simul*).
- Bayesian tests of independence and model selection for loglinear models (1990, 1996 *Canad. J. Statist.*).
- Bayesian detection of outlying counts in a table that deviate from the independence model (1997 *JASA*).
- Bayesian fitting of binary and multinomial models (with S. Chib, (1993 *JASA*, $>$ 1000 citations)
- Book *Ordinal Data Modeling* (1999, with V. Johnson)

## Contributions by UF-WW participants: Jon Forster

- Monte Carlo methods for exact conditional inference in loglinear and logistic models (1996 *JRSS-B*, *JRSS-A*, 1999 *Biometrics*, 2003 *Statistics and Computing*)
- Bayesian model and variable selection using MCMC (2000 J. Stat. Comput. Simul., 2002 *Statistics and Computing*, 2003 *JSPI*, 2007 *Comput. Statist. & Data Anal.*)
- Bayesian analysis of binary crossover data (1994 *Statistician*)
- *Kendall's Advanced Theory of Statistics, vol. 2b: Bayesian Inference* (2004) with J. K. O'Hagan, has chapter on categorical data analysis
- Ongoing work on Bayesian sensitivity analysis for missing multivariate categorical data, Bayesian conjugate inference and Jeffreys' priors for loglinear models

# Contributions by UF-WW participants: Gary Koch

- WLS methods for modeling categorical response variables; e.g., with GSK (1969 *Biometrics*) and analyzing repeated measurement data (1977, Biometrics)
- Randomization-based methods as extensions of the Cochran-Mantel-Haenszel test (e.g., 1978 *ISR*, 1988 *Ann. Rev. Public Health*) randomization-based analysis of covariance, often in a randomized clinical trials setting (e.g., 1982 *Biometrics* and 2005 *Statist. Methods Medic. Res.*).
- Asymptotic covariance structure for estimated parameters from nonstandard loglinear models, such as raked tables (e.g., 1981 *ISR*).
- Measuring inter-rater agreement (with Landis, 1977 *Biometrics* $>$ 11,000 citations)

## Contributions by UF-WW participants: Diane Lambert

- Nonparametric mixtures of Poisson distributions, zero-inflated Poisson mixture model (1984 *Annals Statist.*, 1992 *Technometrics* $>$ 750 citations)
- Nonparametric mixtures of logistic regression models (1989 *JASA*)
- Overdispersion diagnostics for generalized linear models (1995 *JASA*)
- Data confidentiality issues (1996 *JASA*)
- Monitoring streams of network counts (2001, 2006 *JASA*)
- Estimation of rare binary events in search engines (Google)

# Contributions by UF-WW participants: Joe Lang

- ML inference for general multinomial/Poisson constraint-model class $\mathbf{h}(\boldsymbol{\mu}) = \mathbf{0}$ that include awkward-to-fit models such as marginal models (1994, 1999, 2005 *JASA*, 1996, 2004, 2006 *Ann. Statist.*, 1995 *JRSS-A*)
- Clarifying equivalences and differences between Poisson and multinomial models (1996 *JRSS-B*)
- Partitioning goodness-of-fit statistics for multivariate categorical models (1996 *JASA*)
- Observed information for loglinear models with incomplete data, e.g. latent class models (1992 *Biometrika*)
- Bayesian ordinal regression with a parametric family of mixture links (1999 *Computat. Statist. & Data Anal.*)
- R functions *mph.fit* for ML fitting of multinomial-Poisson homogeneous models, *ci.table* for score and profile likelihood confidence intervals for categorical parameters

## Contributions by UF-WW participants: Xihong Lin

- Bias correction in PQL estimation in generalized linear mixed models (1995 *Biometrika*, 1996 *JASA*), and variance component testing (1997 *Biometrika*, 1999 *Biometrics*)
- Generalized linear mixed models with measurement error (1998 *JASA*, 1999 *Biometrics*)
- Semiparametric and nonparametric regression for longitudinal data, including generalized additive mixed models, kernel and spline smoothing, profile likelihood methods (1998, 2000, 2001, 2002, 2005, 2006 *JASA*, 2001, 2004, 2008 *Biometrika*, 2003, 2004, 2005, 2007, 2009 *Biometrics*, 1999, 2006 *JRSS-B*)
- Modeling categorical data with high-dimensional covariates using kernel machines (2007 *Biometrics*, 2008 *BMC Bioinformatics*)

## Contributions by UF-WW participants: Stu Lipsitz

- GEE for correlated binary data using the odds ratio and evaluating performance of GEE methods (1991 *Biometrika*, 1994, 1996 *Biometrics*)
- Extending GEE to handle clustered nominal or ordinal data (1994 *Statist. Medic.*, 1995 *JASA*)
- Dealing with missing data in categorical data analysis (1994, 1996, 1998, 1999, 2001 *Biometrics*, 1995, 1999 *JRSS-B*, 1996, 1998, 2001 *Biometrika*, 1992, 1997, 1998, 2000 *JRSS-C*, 1999, 2005 *JASA*, 1999 *Statist. Medic.*)
- Miscellaneous topics, such as goodness of fit of ordinal regression models (1994 *Statist. Medic.*, 1996 *JRSS-C*), binary time series data (1995 *JRSS-C*), ML methods for non-standard models (1990 *Statist. Medic.*, 1992 *Biometrics*, 1990, 2002 *Biometrika*), modeling agreement (1994, 2000, 2003 *JRSS-C*, 2000 *Biomet. J.*, 2001 *JRSS-A*)

# Contributions by UFWW participants: Peter McCullagh

- *Generalized Linear Models* with J. Nelder (2nd ed., 1989, $>$ 15,000 citations)
- Modeling ordinal data with arbitrary link for cumulative probabilities, extended model allowing dispersion (1980, 1984 *JRSS-B*)
- Consistency, asymptotic normality, and efficiency for for quasi likelihood estimates (1983 *Ann. Statist.*)
- Asymptotic theory for goodness-of-fit statistics (1985 *ISR*, 1986 *JASA*)
- Inference in GLMs and GLMMs (1990, 1991, 1995 *JRSS-B*, 1993 *Biometrics*, 1994 *Ann. Statist.*, 1994 *Biometrika*)
- Papers on fundamentals: What is a statistical model? (2002 *Ann. Statist.*), invariance and factorial models (2000 *JRSS-B*), sampling bias and logistic models (2008 *JRSS-B*)

## Contributions by UF-WW participants: Art Owen

- Substantial research work on empirical likelihood; e.g., heavily cited articles on empirical likelihood-ratio confidence regions (1988 *Biometrika*, 1990 *Ann. Statist.*), empirical likelihood for linear models (1991 *Ann. Statist.*), overview in 2001 Chapman & Hall text *Empirical Likelihood*.
- Infinitely imbalanced logistic regression, alternative (unbalanced) asymptotics (2006 *J. Machine Learning Research*)
- Quasi-Monte Carlo sampling in many recent papers and tech reports listed at www-stat.stanford.edu/ owen/reports
- Recent work on resampling methods for matrix valued data, motivated by bioinformatics and internet data.
- Consulting about and teaching categorical data methods

# Contributions by UF-WW participants: Nancy Reid

- Parameter orthogonality and approximate conditional inference (1987 *JRSS-B*)
- Saddlepoint methods (1988 *Statist. Sci.*) and conditioning in inference (1995 *Statist. Sci.*)
- Wald lecture on asymptotics and the theory of inference (2003 *Ann. Statist.*)
- Improved likelihood inference for discrete data (2006 *JRSS-B*)
- Books *Applied Asymptotics: Case Studies in Small-Sample Statistics* with A. Davison and A. Brazzale (2007), *Theory of Design of Experiments* with D. R. Cox (2000)
- Overview of composite likelihood methods (2009 *Statist. Sinica*)

## Some emerging themes

- High dimensional problems
- Genomics and other areas of computational biology
- Nonparametric and semiparametric approaches
- Bayesian CDA for models with large numbers of parameters
- Diagnostics for Bayesian analyses and hierarchical models such as GLMMs
- Non-model-based methods for classification and prediction such as CART and other methods (e.g., Hastie, Tibshirani, Friedman *The Elements of Statistical Learning*)
- Others we'll learn about at this workshop!

# APOLOGY

Sorry about all the contributions I've not mentioned
(or just do not know about yet)!

Your comments, insights, additions to the story, are welcome!