

STA 6934 Topics in Basic Analysis

CLASS: MWF Period 8 (3:00 p.m.- 3:50 p.m.) FLO 230

INSTRUCTOR:

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OFFICE HOURS: Period 6 (MW) and Period 7 (MWF)

OBJECTIVE: The objectives of this course are:

- (i) to provide prospective statistics doctoral students with the *knowledge* of basic mathematical analysis which is needed for the study of measure-theoretic probability (STA 7466-7467), and
- (ii) to enable prospective statistics doctoral students to reach a level of *mathematical sophistication* needed for the study of measure-theoretic probability (STA 7466-7467).

Although this course can count as a masters elective, it is appropriate for terminal masters students only if either they anticipate doctoral work in statistics sometime ahead in the future or they have a genuine interest in learning basic mathematical analysis as a component of their overall education. In general, terminal masters students should register instead for a statistics elective course of an applied nature.

We will probably not be able to cover every topic from mathematical analysis which ideally should be known prior to the study of measure-theoretic probability. However, by completing this course, students should have developed the mathematical maturity to enable them to read on their own about any needed topic which was not covered.

It would *not* be appropriate to think of this course as “Calculus Four” or “Advanced Calculus”. Indeed, it would be better to think of it as “Calculus Zero” or “Calculus Infinity” since we will be mainly concerned with foundational matters and the underlying theory of calculus rather than with developing a collection of methods and techniques. Precise definitions and theorems (and their proofs) form the main components of the course. Examples and counterexamples are also provided to elucidate the fine and subtle points of the general theory. Emphasis is placed on rigorous development of mathematical concepts and the development of critical thinking skills.

PREREQUISITES: As a minimum, three semesters of honest calculus are needed. Additional work in mathematics would of course be desirable but strictly speaking not necessary.

COURSE MATERIAL:

- (i) STA 6934 Course Notes (available from Target Copy on University Avenue)
- (ii) Kenneth A. Ross, *Elementary Analysis: The Theory of Calculus*. Springer, New York (This book is optional and is definitely not required.)

We will follow the Course Notes very carefully. The Ross text will only serve as a general reference. We will not cover every topic in the Ross text and we will also cover some topics which are not in the Ross text. As a rough guide, we will attempt to cover much of Chapters 1 to 4 and touch on parts of Chapters 5 and 6 of the Ross text. However, while the treatment of many of the topics covered will be identical to that in the Ross text except for minor modifications, the treatment of other topics covered will be substantially different from that given in the Ross text.

It may be mentioned that by far the most famous textbook containing the material presented in this course is the textbook: Walter Rudin, *Principles of Mathematical Analysis*, 3rd edition. McGraw Hill, New York. This textbook is designed for a two semester course sequence. It has remained a top seller ever since the first edition appeared over 55 years ago! The reasons that the Ross text was chosen as the general reference rather than the Rudin text are that (i) the Ross text is basically intended for a one semester course, and (ii) the Ross text is far more “student friendly” than the Rudin text.

TOPICS: (Chapter numbers refer to the STA 6934 Course Notes)

Chapter 1. Review of set theory, the well-ordering property of the natural numbers and the principle of mathematical induction, the real numbers \mathbf{R} with emphasis on the completeness axiom, supremum and infimum of a subset of \mathbf{R} , the Archimedean Property of \mathbf{R} . (Approximately 4 to 5 lectures)

Chapter 2. Sequences in \mathbf{R} , convergence, infinite limits, monotone sequences, limit superior and limit inferior of a sequence and their relation to the limit of a sequence, Cauchy sequences, subsequences, the Bolzano-Weierstrass theorem, metric space topology with emphasis on Euclidean k spaces, open and closed sets, accumulation points, compactness, the Heine-Borel theorem, infinite series, alternating series, rearrangement of series. (Approximately 14 to 15 lectures)

Chapter 3. Limits of functions, continuity of functions, properties of continuous functions, uniform continuity. (Approximately 7 lectures)

Chapter 4. Sequences and series of functions, convergence and uniform convergence of sequences and series of functions. (Approximately 2 to 3 lectures)

Chapter 5. Selected topics from differential calculus. (Approximately 2 to 3 lectures)

Chapter 6. Selected topics from the theory of the Riemann integral, definition of the Riemann-Stieltjes integral. (Approximately 8 lectures)

The above topics contain results obtained by some of the 19th century’s greatest mathematicians. Measure-theoretic probability (which was developed in the 20th century) could not have been developed without the above topics. We acknowledge that it might not be transparent at the present time that knowledge and genuine understanding of the above topics are a basic and crucial component of a statistics doctoral program. This will be transparent next year in measure-theoretic probability (STA 7466-7467) as well as in some other advanced statistics courses.

It may be surprising that not much time will be spent on Chapter 5 (Differentiation) and Chapter 6 (Integration). Experience has shown that prior to the beginning of this course, students are already rather proficient with differentiation and (Riemann) integration concepts but have a superficial understanding of the foundational material in Chapters 1 to 4. Thus we emphasize the material in Chapters 1 to 4. Moreover, in STA 7466 we will develop and use another form of integration (the Lebesgue integral) which has nicer properties than the Riemann integral of calculus.

POLICIES: Course grades will be determined by performance on six exams, each of which is worth 50 points, giving a total of 300 possible points.

- Exam 1 (in class, closed book and notes) Chapter 1 and Chapter 2 (up to page 48 of Course Notes)
- Exam 2 (take-home, you may refer to your Course Notes and the Ross text only)
Chapter 1 and Chapter 2 (up to page 48 of Course Notes)
- Exam 3 (in class, closed book and notes) Chapter 2 (starting with page 49 of Course Notes and Chapter 3)
- Exam 4 (take-home, you may refer to your Course Notes and the Ross text only)
Chapter 2 starting with page 49 of Course Notes and Chapter 3)
- Exam 5 (take-home, you may refer to your Course Notes and the Ross text only)
Emphasis on Chapters 4,5,6
- Exam 6 (in class, closed book and notes) Emphasis on Chapters 4,5,6

The “in class” exams (Exams 1, 3, and 6) will consist of giving precise definitions, stating theorems, solving some “not too difficult” problems (proofs required) and providing examples/counterexamples. The “take-home” exams (Exams 2, 4, and 5) will consist of solving problems (proofs required) and providing examples/counterexamples. The “take-home” exam questions might be more difficult than the “in class” exam questions, or they might be of the same level of difficulty but take longer to write out the solution.

Exams 1 and 2 will occur at approximately the same time; similarly for Exams 3 and 4. The Exam 5 (take-home) will be handed out on Friday April 20, 2018 and is due on Wednesday April 25, 2018 at the beginning of class. The Exam 6 (in class) will be on Wednesday April 25, 2018.

ATTENDANCE: Classroom attendance is fully expected.

ACADEMIC HONESTY : University of Florida students are expected to abide by the following: “We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honesty and integrity.”

CLASSROOM ACCOMMODATIONS: Students requesting classroom accommodations must first register with the Disability Resource Center. The Disability Resource Center will provide the student with a letter to be given to the instructor when requesting accommodations.

GRADING SCALE:

A= 270-300 points

B= 240-269 points

C= 210-239 points

D= 180-209 points

E= 0-179 points

In any case, the numerical scores required for the various letter grades will not be higher than those given above (and typically they will be lower). Plus or minus grades can be assigned in borderline cases.

INFORMATION ON CURRENT UF GRADING POLICIES FOR ASSIGNING GRADE POINTS:

See <https://catalog.ufl.edu/ugrad/current/regulations/info/grades.aspx>.

STUDENT EVALUATION OF INSTRUCTION:

Students will have the opportunity to provide feedback on the quality of instruction in this course and all students are expected to complete the online evaluation which will be available near the end of the semester.