STA 7466 | **Probability Theory I** | **Fall, 2018**
Section 5956 | | **Class No. 20220**

Course Information

Time: MWF 3:00 – 3:50 p.m. (per. 8) **Location:** FLO 230 (Griffin-Floyd Hall)

Instructor: Dr. Brett Presnell

Office: FLO 225 **E-mail:** `presnell@stat.ufl.edu`

Office Hours: See instructor's web page. **Phone:** 352-273-2989

Web Page: `http://www.stat.ufl.edu/~presnell`

Recommended Text: Billingsley (1995). *Probability and Measure* (3rd ed). Wiley, New York.

Course Content and Objectives

The two-semester sequence STA 7466–7467 covers material from measure, integration, and probability theory that every statistics doctoral student should know. This includes a rigorous development of measure theory and Lebesgue integration, independence, modes of convergence of random variables, convergence of series of independent random variables, weak and strong laws of large numbers, characteristic functions, the central limit theorem, conditional expectation, basic martingale theory, and the Wiener process (a.k.a., Brownian motion). Other topics will be introduced as time permits.

Grading

Attendance will account for 5% of the course grade. Each student is allowed up to three unexcused absences; any additional absences will be excused only if they are documented and conform to the attendance policies of the Graduate School as described in the Graduate Catalog. If you know that you will have to miss class for an excused reason, please inform the instructor in advance of your absence.

Homework will be collected regularly throughout the term and will determine 20% of the course grade. Late homework will not be accepted.

There will be two in-class exams, each accounting for 20% of the course grade, and a final exam accounting for 35% of the course grade. Make up exams will be given only in case of an excused absence. All exams will be in-class. No books, notes, or other reference materials will be allowed during the exams.

Prerequisites

Students are assumed to have experience with and a working knowledge of basic set theory and elementary classical real analysis. Suitable prerequisites are STA 6394 Topics in Basic Analysis or MAA 5228 Modern Analysis I. Students fluent in most of the topics in a text such as Rudin's *Principles of Mathematical Analysis* (first 7 chapters) should be well prepared to enter this course.

Supplementary References

There are many wonderful books on probability theory, although most implicitly or explicitly assume that you already know some measure theory. You may find it useful to consult any in the following list.

1. Athreya, K. B. and Lahiri, S. N. (2006). *Measure Theory and Probability Theory.* Springer, New York. (A new book. I haven't had a chance to read it carefully, but it looks very similar to my notes. I may adopt it as the course text in the future.)

2. Brieman, L. (1968). *Probability.* Addison-Wesley, Reading, Mass. (A classic, now available in a reprint edition from SIAM.)

3. Chung, K. L. (1974). *A Course in Probability Theory* (2nd ed.). Academic Press, New York. (The text I had as a student, also a classic. There is now a second editon.)

4. Dudley, R. M. (1989). *Real Analysis and Probability.* (Originally published by Chapman & Hall, then Wadsworth, then CRC, and now by Cambridge. An excellent book with more of a "topological" emphasis. The paperback version from Cambridge is a bargain.)

5. Durrett, R. (2004). *Probability: Theory and Examples* (3rd ed.). Duxbury Press, Belmont, California. (Many good examples.)

6. Feller, W. (1966). *An Introduction to Probability Theory and Its Applications*, Vol. II. Wiley, New York. (Very "analytical," but has certain things that are hard to find anywhere else.)

7. Fristedt, B. and Gray, L. (1997). *A Modern Approach to Probability Theory.* Birkhäuser, Boston.

8. Karr, A. F. (1993). *Probability.* Springer-Verlag, New York. (More elementary than the rest in this list.)

9. Loeve, M. (1955, 1st ed). *Probability Theory.* Van Nostrand, Princeton, New Jersey. (This is a real classic, and there are many editions. Your (great?) grandfather learned probability from this book.)

10. Pollard, D. (2001). *A User's Guide to Measure Theoretical Probability.* Cambridge University Press. (A new book, written in an interesting and lively style; inexpensive in paperback.)

11. Resnick, S. I. (1999). *A Probability Path.* Birkhäuser, Boston. (Brief and to the point, with lots of good exercises, but it's new, so beware of the occassional misprint or mistake.)

12. Shorack, G. R. (2000). *Probability for Statisticians.* Springer-Verlag, New York. (Interesting new book, but again, beware of misprints and mistakes.)

13. Shiryayev, A. N. (1996, 2nd ed). *Probability.* Springer, New York.

Guidelines for Homework and Exams

(with thanks to Ian McKeague)

1. Start each problem on a separate sheet of paper; write your name at the top right-hand corner of the first sheet. If a problem continues over several pages, write *(continued)* at the bottom of the page and write the problem number and *(continued)* at the beginning of the next page.

2. Write neatly and legibly. Do not be overly concerned about saving paper: write only on the front side of each page, do not crowd your writing, and make it large enough so that it can be read without eyestrain.

3. Mathematics is prose. Each statement should be a sentence, generally with a subject, object, and verb. End an equation with a punctuation mark if it is at the end of a sentence. An $=$ sign can operate as a verb. Never start a sentence with a mathematical symbol or other notation.

4. Do not use unnecessary words—use notation to cut down on tedious repetition.

5. Do your exploratory work on scratch paper and do NOT turn it in with your final solution. If you are asked to prove something for all finite n, special cases (e.g., $n = 1$, $n = 2$) are considered exploratory, unless they are the beginning of an induction argument.

6. The Good Samaritan Rule: when you need to use a standard result, mention its *name*, and not a theorem number. If the result has no name, but appears in the textbook or course notes, then you may refer to it by number. Otherwise, you should state the result, at least in outline (and include a proof if it is not a standard result from class or from real analysis). Don't assume the reader knows what you are about to do—it is often helpful to outline the steps of your solution before plunging into details.

7. For homework problems, write out the question before giving the solution. Answer the problems in the order in which they were assigned. Staple the sheets of paper together (and do not write near the upper left-hand corner of the page where the staple will go).

8. If you introduce some notation which was not specified in the problem, you must define or specify it. A common mistake is to use an ϵ without initially saying "Let $\epsilon > 0$."

9. Your work will be more readable if you use displayed equations rather than embedding long equations in the text.

10. Each step of your solution needs to be justified, either by naming a standard result, or filling in the gap by a separate argument. If you are unable to fill the gap (or do any part of the problem), say so explicitly; this is far better than writing down a specious argument.

11. If you are stuck on a homework problem, ask me for a hint. You have nothing to lose by asking for a hint, but you do have something to lose by handing in incomplete work.

12. Do not copy from others. Your solution must reflect your own understanding of the problem, not that of someone else.

Course Outline

The following schedule is tentative and subject to change. The pace of the course will be rigorous and students should plan on several hours of outside study for every hour spent in class.

- `[2018-08-22 Wed]` Measurable Spaces
- `[2018-08-24 Fri]` Measures
- `[2018-08-27 Mon]` Extension of Measures: Uniqueness
- `[2018-08-29 Wed]` Extension of Measures: Existence
- `[2018-08-31 Fri]` Lebesgue and Lebesgue-Stieljes Measures
- `[2018-09-03 Mon]` Labor Day (NO CLASSES)
- `[2018-09-05 Wed]` Measurable Functions
- `[2018-09-07 Fri]` Approximation by Simple Functions
- `[2018-09-10 Mon]` Induced Measures and Distributions
- `[2018-09-12 Wed]` Lebesgue Integration
- `[2018-09-14 Fri]` Properties of the Integral
- `[2018-09-17 Mon]` Limits and Integration
- `[2018-09-19 Wed]` Limits and Integration (ctd)
- `[2018-09-21 Fri]` Catch up and review (END OF MATERIAL FOR EXAM 1)
- `[2018-09-24 Mon]` Change of Variables
- `[2018-09-26 Wed]` Comparison of Reimann and Lebesgue Integrals
- `[2018-09-28 Fri]` The Radon-Nikodym Theorem
- `[2018-10-01 Mon]` EXAM 1
- `[2018-10-03 Wed]` The Hahn and Jordan Decompositions
- `[2018-10-05 Fri]` Proof of the Radon-Nikodym Theorem
- `[2018-10-08 Mon]` Singular measures
- `[2018-10-10 Wed]` Product Measure Spaces
- `[2018-10-12 Fri]` Fubini's Theorem
- `[2018-10-15 Mon]` Examples
- `[2018-10-17 Wed]` Infinite Products of Measure Spaces
- `[2018-10-19 Fri]` L^p Spaces

- [2018-10-22 Mon] Inequalities for Expectations
- [2018-10-24 Wed] Modes of Convergence
- [2018-10-26 Fri] Catch Up and Review (END OF MATERIAL FOR EXAM 2)
- [2018-10-29 Mon] Modes of Convergence (ctd)
- [2018-10-31 Wed] Uniform Integrability
- [2018-11-02 Fri] Homecoming (NO CLASSES)
- [2018-11-05 Mon] EXAM 2
- [2018-11-07 Wed] Independent Events and Independent Classes
- [2018-11-09 Fri] Independent Random Variables
- [2018-11-12 Mon] Veteran's Day (NO CLASSES)
- [2018-11-14 Wed] Borel-Cantelli Lemmas and Kolmogorov's Zero-One Law
- [2018-11-16 Fri] Maximal Inequalities
- [2018-11-19 Mon] Convergence of Random Series
- [2018-11-21 Wed] Thanksgiving break (NO CLASSES)
- [2018-11-23 Fri] Thanksgiving break (NO CLASSES)
- [2018-11-26 Mon] Convergence of Random Series (ctd)
- [2018-11-28 Wed] Strong Laws of Large Numbers
- [2018-11-30 Fri] Strong Laws of Large Numbers (ctd)
- [2018-12-03 Mon] Glivenko-Cantelli Theorem
- [2018-12-05 Wed] Weak Laws of Large Numbers
- [2018-12-10 Mon] FINAL EXAM (10 AM –12 PM)