STA 6276 Statistical Computing II: Monte Carlo Methods Spring 2025

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Class Time MWF Period 3 (9:35 am–10:25 am)

Office Hours MWF Period 4 (10:30 am–11:30 am)

Course Description Monte Carlo methods are now used in virtually every scientific area, including statistical physics (where they originated), Bayesian and frequentist statistical inference, image reconstruction, and various parts of machine learning. The basic idea is to carry out a simulation to estimate quantities of interest that cannot be computed analytically. This course will begin with a brief discussion of some standard Monte Carlo schemes, before moving to Monte Carlo methods based on Markov chains.

Consider the situation where there is a distribution π on some space, and we are interested in estimating π or $\int f d\pi$ where f is some function, but π is analytically intractable. Markov chain Monte Carlo proceeds as follows. We set up a Markov chain with the property that its transition function has π as its stationary distribution. Then we run a chain X_1, X_2, \ldots with this transition function. If the Markov chain converges to its stationary distribution (i.e. for large n, the distribution of X_n is approximately π), then by running the chain long enough, we can obtain a sample from π . This sample can be used to estimate π or some feature of it such as $\int f d\pi$.

In this course we will understand the method in detail, describe the main implementations, discuss some large classes of problems in statistics where it has had success (primarily in Bayesian inference), and apply the method on several real data sets. The method is not fool-proof. We will discuss some of the mathematical results pertaining to convergence issues, and also discuss some practical convergence diagnostics.

Course Web Page All course related material will be posted on Canvas.

Prerequisites/Corequisites STA 6326 (Introduction to Theoretical Statistics I) is a prerequisite and STA 6327 (Introduction to Theoretical Statistics II) is a corequisite. You also need to know some probability theory beyond what is covered in the Master's program, but we will go over these facts, as needed, in the class. Additionally, you need to be familiar with the statistical computing language R. No knowledge of Markov chains is assumed. Grading Your course grade will be based on the four components below, with the stated weights.

Exam 1: Wednesday, February 19 (Time and Place TBA) 20%

Exam 2: Wednesday, March 26 (Time and Place TBA - Covers the material after Exam 1) 20%

Exam 3: Wednesday, April 23 (Time and Place TBA - Covers the material after Exam 2) 20%

HW: There will be about 6 homeworks assigned during the semester 40%

Some of the homework assignments will be of a theoretical nature, and some will involve computer implementation of the methods we discuss on specific data sets. The solutions to the homework assignments must be entirely your own (this applies also to the R code).

Topics

- Review of Maximum Likelihood Estimators and their asymptotic properties
- Issues in practical implementation of Bayesian statistics
- Illustrative example: censored data
- Basic Monte Carlo methods
- General idea of Markov chain Monte Carlo
- The Gibbs sampler (general properties; application to latent variable models, including hierarchical Bayesian models and censored data models; application to high-dimensional problems)
- Rao-Blackwellization and variants thereof
- Convergence diagnostics
- The Metropolis-Hastings algorithm (general properties; application to Ising model; random walk chains and independence chains; adaptive rejection Metropolis sampling)
- Hamiltonian Monte Carlo
- Theory of convergence (ergodic theorems and central limit theorems)
- An additional in-depth application of MCMC to some nontrivial area in statistics, which is likely to be either Bayesian variable selection in regression or latent Dirichlet allocation in topic modelling

General Course Policies

- Cell phones may not be used; they should be turned off (or set on silent). Laptops must be shut.
- If you have a disability and will request accommodations, please see me as early in the semester as possible.