Probability Theory I

Fall 2021

STA 7466

Course Information

Time: MWF 8:30 am - 9:20 am (per. 2) Location: FLO 230 (Griffin-Floyd Hall)

Instructor: Dr. Kshitij Khare

Office: FLO 208 E-mail: kdkhare@stat.ufl.edu

Office Hours: MWF 9:30 am - 10:30 am, FLO 208 Phone: 352-273-2985

Recommended Text

Billingsley (1995). Probability and Measure (3rd ed). Wiley, New York.

Course Files and Other Information

Access in Canvas by logging into https://elearning.ufl.edu

Course Content and Objectives

The two-semester sequence STA 7466–7467 covers material from measure, integration, and probability theory that every statistics doctoral student should know. This includes a rigorous development of measure theory and Lebesgue integration, independence, modes of convergence of random variables, convergence of series of independent random variables, weak and strong laws of large numbers, characteristic functions, the central limit theorem, conditional expectation, basic martingale theory, and the Wiener process (a.k.a., Brownian motion). Other topics will be introduced as time permits.

Grading

Attendance will account for 5% of the course grade. Each student is allowed up to three unexcused absences; any additional absences will be excused only if they are documented and conform to the attendance policies of the Graduate School as described in the Graduate Catalog. If you know that you will have to miss class for an excused reason, please inform the instructor in advance of your absence.

Homework will be collected regularly throughout the term and will determine 35% of the course grade. Late homework will not be accepted.

There will be three in-class exams, each accounting for 20% of the course grade. Make up exams will be given only in case of an excused absence. All exams will be in-class. No books, notes, or other reference materials will be allowed during the exams.

Prerequisites

Students are assumed to have experience with and a working knowledge of basic set theory and elementary classical real analysis. Suitable prerequisites are STA 6394 Topics in Basic Analysis

or MAA 5228 Modern Analysis I. Students fluent in most of the topics in a text such as Rudin's *Principles of Mathematical Analysis* (first 7 chapters) should be well prepared to enter this course.

Course Evaluations

Students are expected to provide professional and respectful feedback on the quality of instruction in this course by completing course evaluations online via GatorEvals. Guidance on how to give feedback in a professional and respectful manner is available at https://gatorevals.aa.ufl. edu/students/. Students will be notified when the evaluation period opens, and can complete evaluations through the email they receive from GatorEvals, in their Canvas course menu under GatorEvals, or via https://ufl.bluera.com/ufl/. Summaries of course evaluation results are available to students at https://gatorevals.aa.ufl.edu/public-results/.

Course Outline

We will cover at least the first four chapters in the course notes. The pace of the course will be fairly rigorous and students should plan on several hours of outside study for every hour spent in class.

Supplementary References

There are many wonderful books on probability theory, although most implicitly or explicitly assume that you already know some measure theory. You may find it useful to consult any in the following list.

- 1. Athreya, K. B. and Lahiri, S. N. (2006). *Measure Theory and Probability Theory*. Springer, New York. (I haven't read this one carefully, but it looks similar in coverage and philosophy to my notes.)
- 2. Brieman, L. (1968). *Probability*. Addison-Wesley, Reading, Mass. (A classic, now available in a reprint edition from SIAM.)
- 3. Chung, K. L. (1974). A Course in Probability Theory (2nd ed.). Academic Press, New York. (The text I had as a student, also a classic. There is now a second edition.)
- 4. Dudley, R. M. (1989). *Real Analysis and Probability*. (Originally published by Chapman & Hall, then Wadsworth, then CRC, and now by Cambridge. An excellent book with more of a "topological" emphasis. The paperback version from Cambridge is a bargain.)
- 5. Durrett, R. (2004). *Probability: Theory and Examples* (3rd ed.). Duxbury Press, Belmont, California. (The best thing about this book is the wealth of good examples.)
- 6. Feller, W. (1966). An Introduction to Probability Theory and Its Applications, Vol. II. Wiley, New York. (Very "analytical," but has certain things that are hard to find anywhere else.)
- 7. Fristedt, B. and Gray, L. (1997). A Modern Approach to Probability Theory. Birkhäuser, Boston. I like this book, which seems to take a different tack from most in a number of places.
- 8. Karr, A. F. (1993). *Probability*. Springer-Verlag, New York. (More elementary than the rest in this list.)
- 9. Loeve, M. (1955, 1st ed). *Probability Theory*. Van Nostrand, Princeton, New Jersey. (This is a real classic, and there are many editions. Your (great?) grandfather learned probability from this book.)
- 10. Munkres, James R. (2000). *Topology* (2nd ed.). Prentice Hall, Upper Saddle River, NJ. An outlier in this list, but sometimes it really helps to know a bit of point-set topology, and the first four or five chapters of this book provide a nice introduction.
- 11. Pollard, D. (2001). A User's Guide to Measure Theoretical Probability. Cambridge University Press. (A new book, written in an interesting and lively style; inexpensive in paperback.)
- 12. Resnick, S. I. (1999). A Probability Path. Birkhäuser, Boston. (Brief and to the point, with lots of good exercises. The early printings had some occassional misprints and/or mistakes, but these may have been fixed by now.)
- 13. Shorack, G. R. (2000). *Probability for Statisticians*. Springer-Verlag, New York. (Interesting book, but again, early printings had some occassional misprints and/or mistakes.)
- 14. Shiryayev, A. N. (1996, 2nd ed). Probability. Springer, New York.

Guidelines for Homework

(with thanks to Ian McKeague and Brett Presnell)

- 1. Mathematics is prose and *grading of homework will reflect this* (think of it as much as a writing exercise as a problem solving exercise). Each statement should be a sentence, generally with a subject, object, and verb. End an equation with a punctuation mark if it is at the end of a sentence. An = sign (and other relationship symbols) can operate as a verb. *Never start a sentence with a mathematical symbol or other notation.*
- 2. Do not use unnecessary words—use notation to cut down on tedious repetition.
- 3. Do your exploratory work on scratch paper and do NOT turn it in with your final solution. If you are asked to prove something for all finite n, special cases (e.g., n = 1, n = 2) are considered exploratory, unless they are the beginning of an induction argument.
- 4. Write neatly and legibly. Do not be overly concerned about saving paper: write only on the front side of each page, do not crowd your writing, and make it large enough so that it can be read without eyestrain.
- 5. Do not use correction fluid ("Wite-Out"). If you make a mistake, erase it or cross it out and continue. If a large portion of the page contains errors, rewrite it on a new, clean sheet of paper.
- 6. Your work will be more readable if you use displayed equations rather than embedding long equations in the text.
- 7. If you introduce some notation which was not specified in the problem, you must define or specify it. A common mistake is to use an ϵ without initially saying "Let $\epsilon > 0$."
- 8. Each step of your solution needs to be justified, either by naming a standard result, or filling in the gap by a separate argument. If you are unable to fill the gap (or do any part of the problem), say so explicitly; this is far better than writing down a specious argument.
- 9. The Good Samaritan Rule: when you need to use a standard result, mention its *name*, and not a theorem number. If the result has no name, but appears in the textbook or course notes, then you may refer to it by number. Otherwise, you should state the result, at least in outline (and include a proof if it is not a standard result from class or from real analysis). Don't assume the reader knows what you are about to do—it is often helpful to outline the steps of your solution before plunging into details.

10. Format:

- Write your name at the top right-hand corner of the first sheet of paper.
- Answer the problems in the order in which they were assigned.
- Start each problem on a separate sheet of paper.
- Write out the question before giving the solution.
- If a problem continues over several pages, write (continued) at the bottom of the page and write the problem number and (continued) at the beginning of the next page.
- Staple the sheets of paper together and do **not** write near the upper left-hand corner of the page where the staple will go (please keep this in mind on exams as well).

- 11. If you are stuck on a homework problem, ask me for a hint. You have nothing to lose by asking for a hint, but you do have something to lose by handing in incomplete work.
- 12. Do not copy from others. Your solution must reflect your own understanding of the problem, not that of someone else.